Design of a digital PID controller by transposing the analog PID

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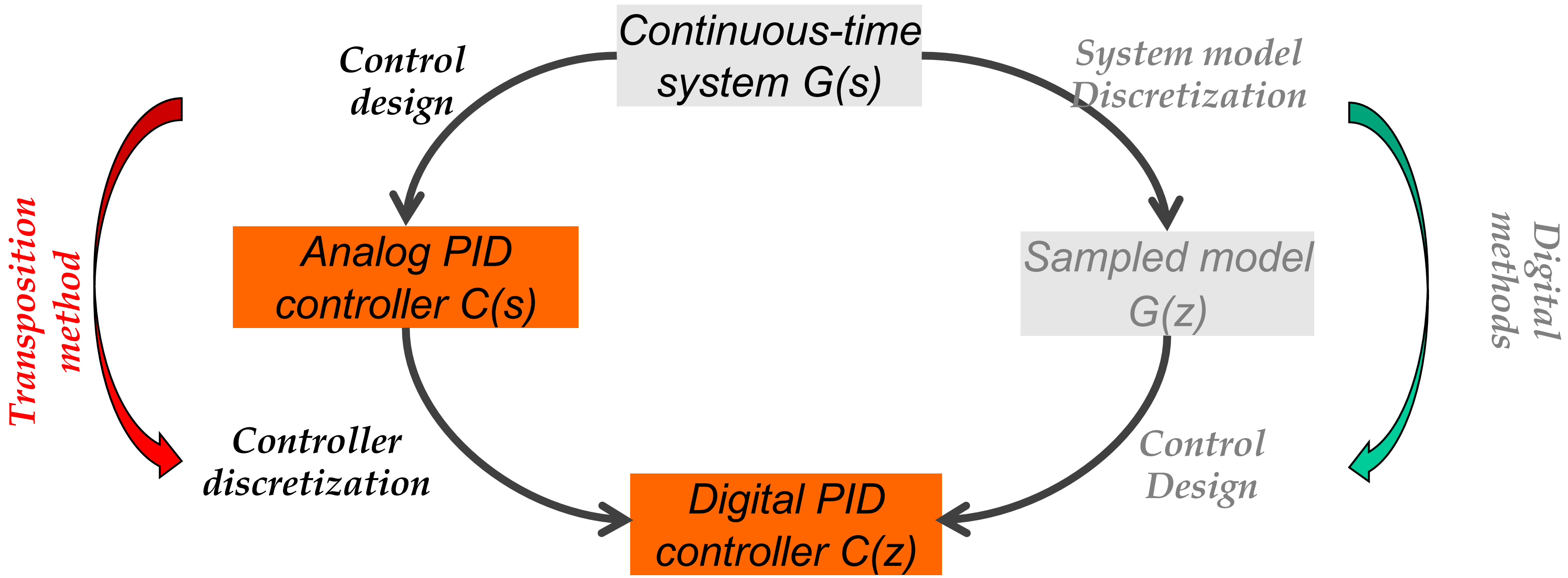
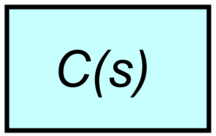
The two paths to digital controller design



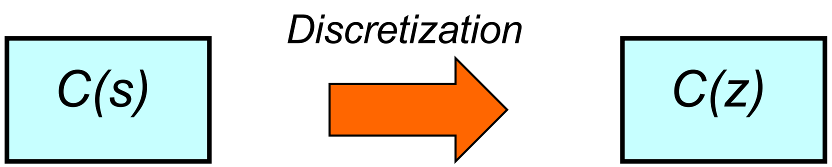
- The design of digital PID controllers by transposing analog PID controllers is an approach commonly used in the industry for two major reasons:
  - analog PID controller design methods are generally well-known
  - performance specifications are easier to interpret with continuous-time models than with sampled models

Digital control design
by transposing the analog controller

- Workflow
  1. Design of a continuous-time PID controller $C(s)$ by one of the traditional design methods (Ziegler-Nichols or others) determined from the continuous-time model



  2. Transpose the continuous-time PID controller $C(s)$ into a digital version $C(z)$ to obtain a digital PID algorithm that behaves as close as possible to the continuous-time control



Parallel PID

In the time domain

$$u(t) = k_p\, \varepsilon(t) + k_i \int_0^t \varepsilon(\tau) d\tau + k_d \frac{\varepsilon(t)}{dt}$$

In the Laplace domain

$$U(s) = \left(k_p + \frac{k_i}{s} + k_d s \right) \varepsilon(s)$$

$$C(s) = k_p + \frac{k_i}{s} + k_d s$$

Ideal PID form encountered in practice

In the time domain

$$u(t) = K_c \left(\varepsilon(t) + \frac{1}{T_i} \int_o^t \varepsilon(\tau) d\tau + T_d \frac{\varepsilon(t)}{dt} \right)$$

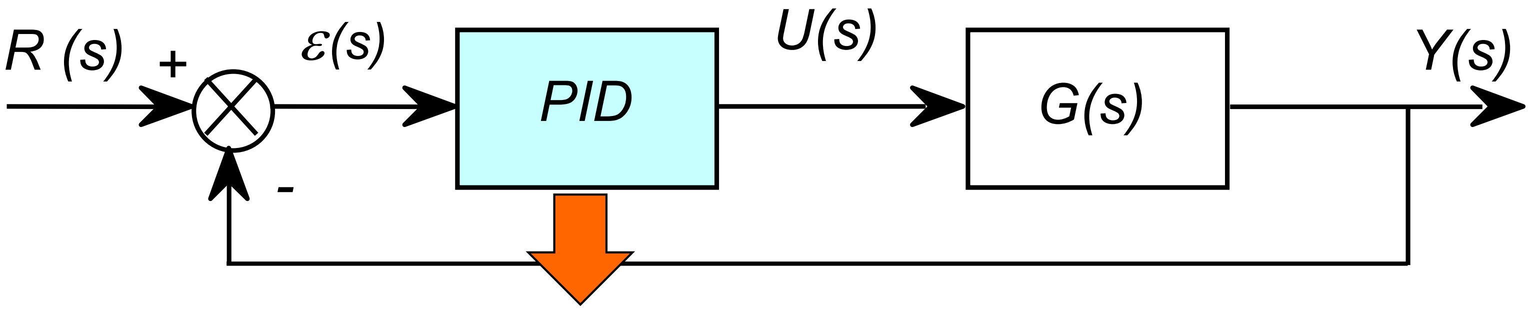
In the Laplace domain

$$U(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \varepsilon(s)$$

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

$$K_p = k_p \qquad T_i = \frac{k_p}{k_i} \qquad T_d = \frac{k_d}{k_p}$$

Reminder about PID controllers



$$u(t) = K_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$

Ideal PID Correcteur $C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$

Real PID Avec dérivée filtrée $C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_d s/N}\right)$ $(N \geq 5)$

Réglage simple et adapté à la plupart des systèmes

- PI : 90%, PID : 95%
- Réglage souvent itératif (essai / erreur)

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Digital version of the PID using common transposition methods

Forward approximation

$$s = \frac{z-1}{T_e} = \frac{1-z^{-1}}{T_e z^{-1}}$$

$$C(z) = K_p\left[1 + \frac{1}{T_i}\frac{T_e z}{z-1} + \frac{N(z-1)}{(1+\frac{NT_e}{T_d})z - 1}\right]$$

Backward approximation

$$s = \frac{z-1}{T_e z} = \frac{1-z^{-1}}{T_e}$$

$$C(z) = K_p\left[1 + \frac{1}{T_i}\frac{T_e}{z-1} + \frac{N(z-1)}{z - (1-\frac{NT_e}{T_d})}\right]$$

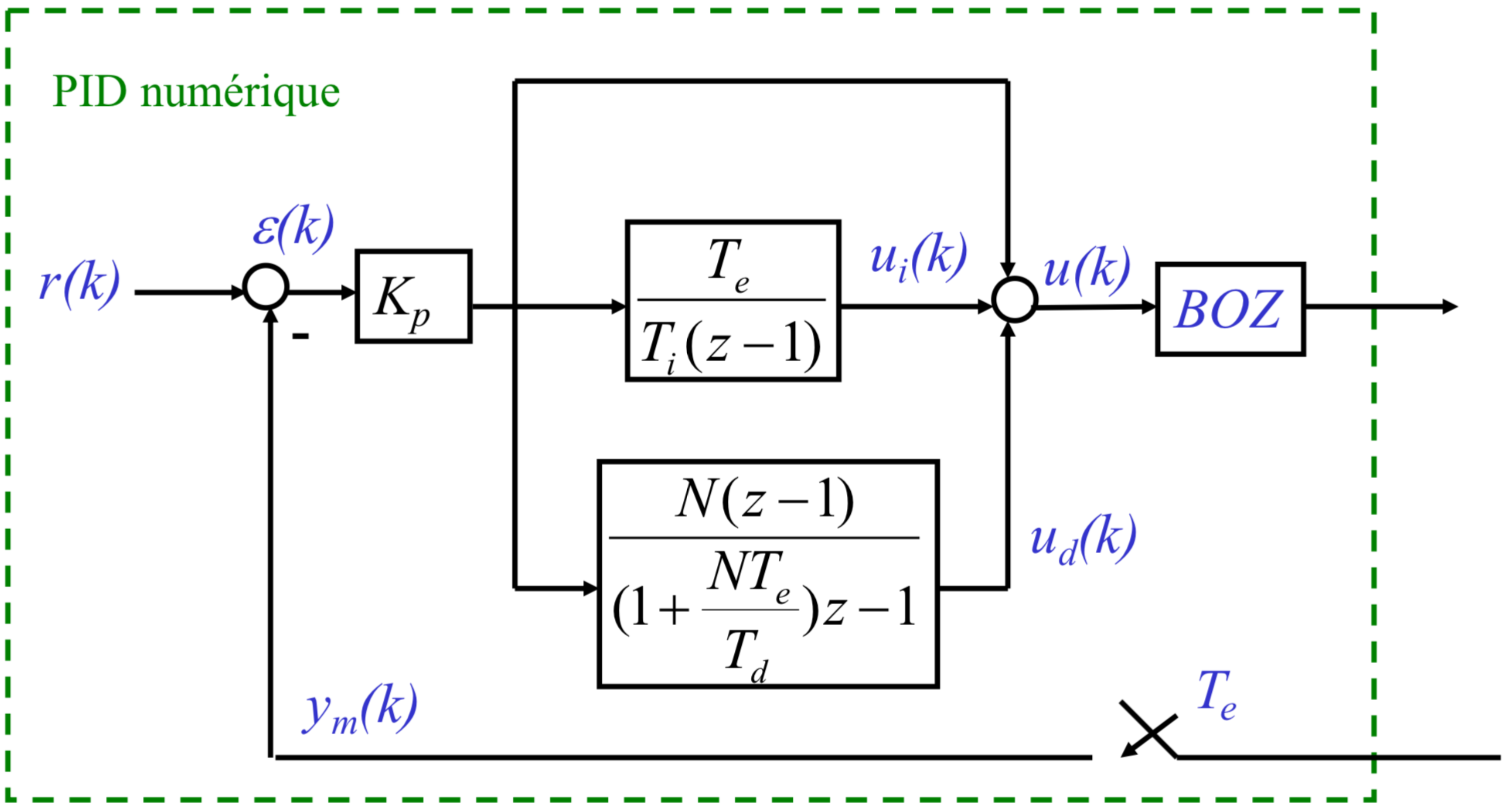
Tustin approximation

$$s = \frac{2}{T_e}\frac{z-1}{z+1} = \frac{2}{T_e}\frac{1-z^{-1}}{1+z^{-1}}$$

$$C(z) = K_p\left[1 + \frac{T_e}{2T_i}\frac{z+1}{z-1} + \frac{N(z-1)}{(1+\frac{NT_e}{2T_d})z - (1-\frac{NT_e}{2T_d})}\right]$$

Backward approximation: *simpler formulas often used in practice for this reason*

Block-diagram of the standard form of a digital PID
(backward approximation)



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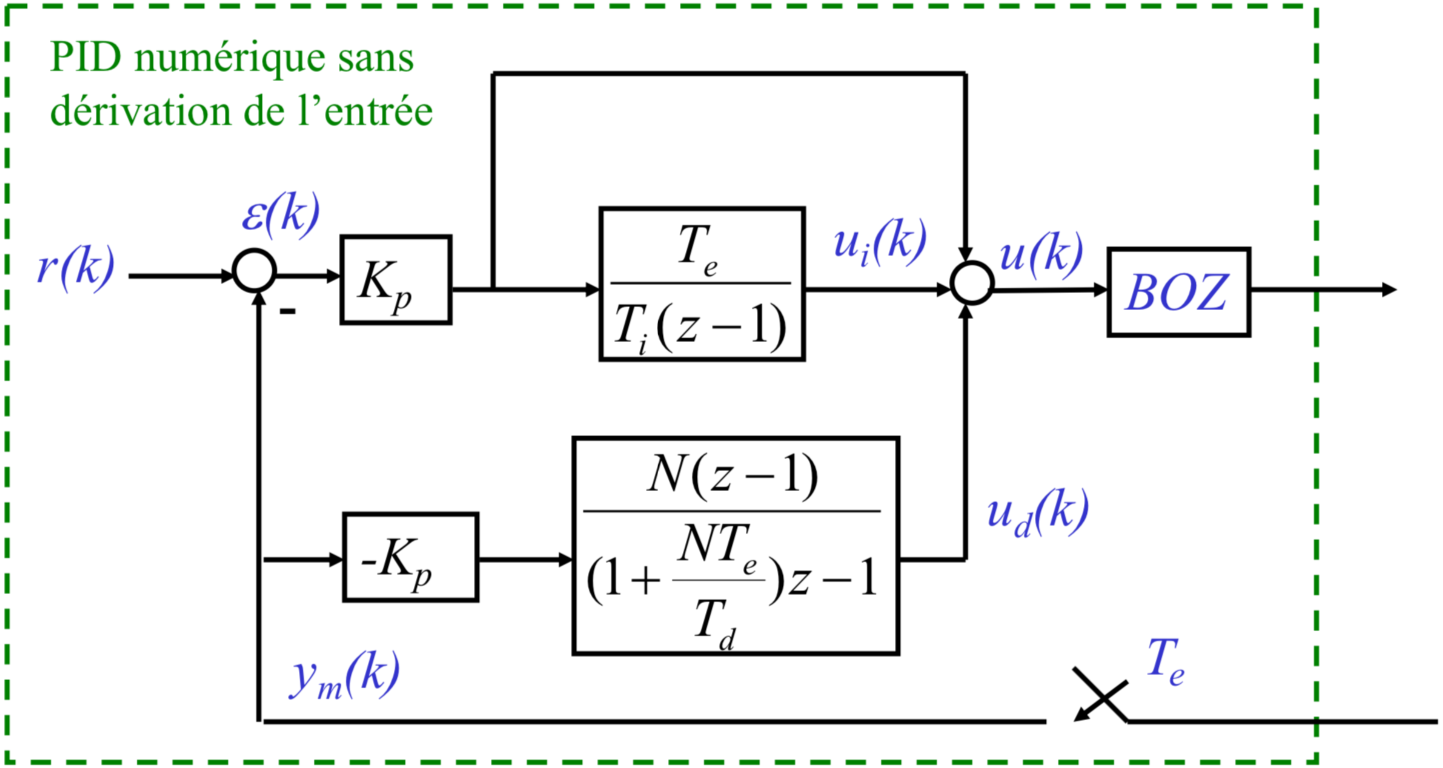
Block-diagram of a digital PID control with the derivative part on the output *(backward approximation)*

In practice, the error term is rarely differentiated to avoid abrupt variations in the control signal when there is a sudden step-like change in the setpoint. The derivative part is applied to the output. The diagram then becomes



Code lines for implementing the digital PID control
(backward approximation)

$$\varepsilon(k) = r(k) - y_m(k)$$

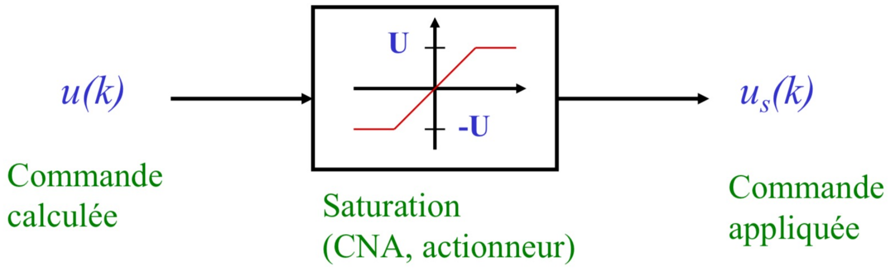
$$u(k) = K_p \varepsilon(k) + u_i(k) + u_d(k)$$

$$u_i(k) = u_i(k-1) + K_p \frac{T_e}{T_i} \varepsilon(k-1)$$

$$u_d(k) = \frac{1}{1 + \frac{NT_e}{T_d}} u_d(k-1) - \frac{K_p N}{1 + \frac{NT_e}{T_d}} [y_m(k) - y_m(k-1)]$$

Anti-windup integral

- The integral part can lead to undesirable effects when, due to an excessively large error, the integrator becomes saturated
- The actuator then remains in the stop position, even when the system output varies

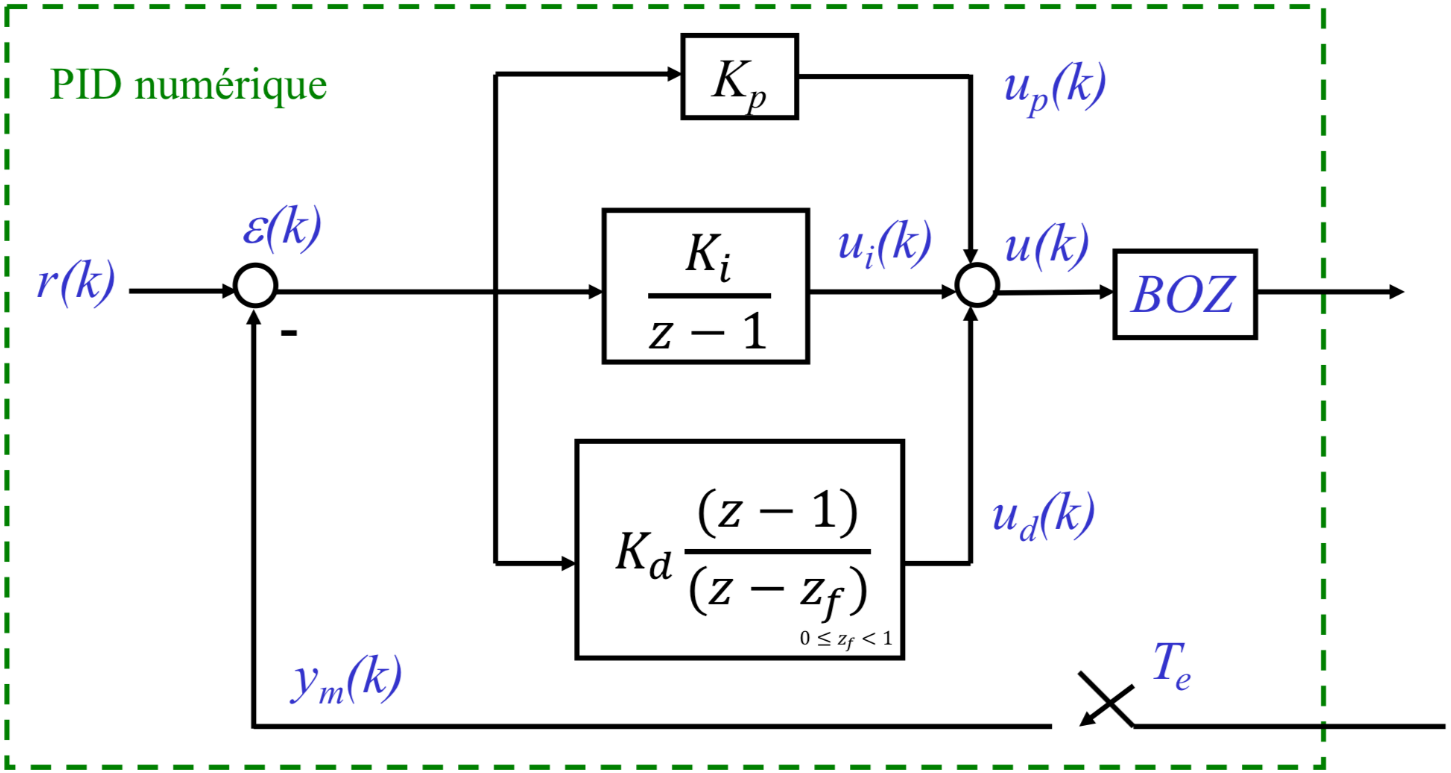


$$u_s(k) = \begin{cases} \mathbf{U} & \text{si} \quad u(k) > \mathbf{U} \\ u(k) & \text{si} \quad |u(k)| \leq \mathbf{U} \\ -\mathbf{U} & \text{si} \quad u(k) < -\mathbf{U} \end{cases}$$

- One possible approach to eliminating this effect is to introduce a loop on the integrator, bringing the difference between the input $u(k)$ and the output $u_s(k)$ of the saturation (real or simulated), with an integration constant T_t

Parallel form of digital PID: more convenient control

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = K_p + \frac{K_i}{s} + K_d s$$



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Digital PID in computer code

```
# def Te, Ki, Kp, Kd, r
i = 0
e0 = e = 0
while running:
    # update measure
    y = ...
    e = r - y
    # update integral
    i += Ki*Te*e
    # compute command
    u = Kp * (e + i + Kd/Te*(e-e0))
    # save current error
    e0 = e
```

Design of the analog PID controller
A brief recap

- To set up the analog PID controller, you need to select

$$K_p\ ,\ T_i\ ,\ T_d\ ,\ T_t\ ,\ N$$

  - N is often set to $N=10$
  - T_t is chosen from the range $[0.1T_i\ ;\ T_i\ ]$

- For the determination of the parameters K_p , T_i , T_d , adjustment methods have been proposed, such as those of Ziegler-Nichols.

- These settings form a starting point that can be refined according to the desired performance

Influence of the P, I and D parts in the case the parallel form of the digital PID

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = K_p + \frac{K_i}{s} + K_d s$$

Gain	T montée	T stabilisation	Dépassement	Erreur statique
$K_p \nearrow$	Diminue	Augmente	Augmente	Diminue
$K_i \nearrow$	Diminue	Augmente	Augmente	Annule
$K_d \nearrow$	–	Diminue	Diminue	–

Démarche : partir d'un premier jeu de gains

- En simulation (système approximé) ou sur système réel
- Préréglage sur réponse indicielle
- Préréglage sur système bouclé

Le préréglage est parfois présenté comme un auto-réglage

Mais souvent c'est vraiment un **pré**réglage...

Digital control by transposition of a analog PID controller – Take away messages

- Recommended transposition methods
  - Bilinear approximation (Tustin) or backward approximation (because of simplest formulas)

- Digital control performance is at best equivalent to that of analog control
  - suitable if the sampling period is fast compared to the main dynamic T_c of the controller: $T_s < T_c /10$

- Even if the stability of the closed-loop system with the analog controller is verified, this does not guarantee the stability of the closed-loop system with the digital controller !
  - In particular, it must be checked that the chosen sampling period does not result in a stability loss of the closed-loop system
  - The effect of the presence of the zero-order hold (additional delay) is not taken into account in the control design and may also affect stability and performance